CALCULATING THE CONVECTION – RADIATION HEATING OF MASSIVE BODIES

Yu. V. Vidin

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We present an iteration method for the calculation of the limit temperature field in thermally massive bodies heated simultaneously by convection and radiation.

The process of heating solid materials to which heat is directed simultaneously by convection and radiation is described, given a number of simplifying conditions, by the equations

$$\frac{\partial\theta(\psi, \mathrm{Fo})}{\partial \mathrm{Fo}} = \frac{\partial^2\theta(\psi, \mathrm{Fo})}{\partial\psi^2} + \frac{\Gamma-1}{\psi}\frac{\partial\theta(\psi, \mathrm{Fo})}{\partial\psi}, \tag{1}$$

$$\frac{\partial\theta(0, \mathrm{Fo})}{\partial\psi} = 0, \tag{2}$$

$$\frac{\partial\theta(1, \mathrm{Fo})}{\partial\psi} = \mathrm{Sk}\left(p[1-\theta(1, \mathrm{Fo})] + 1 - \theta^4(1, \mathrm{Fo})\right), \tag{3}$$

$$\theta(\psi, 0) = \theta_0. \tag{4}$$

Here $p = \mathrm{Bi}/\mathrm{Sk} \geq 0$.

Problem (1)-(4) is classified as nonlinear.

When the heated item is thermally massive, it is comparatively easy to calculate the upper limit of the sought temperature field $\theta(\psi, \mathrm{Fo})$. In this case, the divergence between the boundary temperature $\theta_{\lim}(\psi, \mathrm{Fo})$ and the actual temperature $\theta(\psi, \mathrm{Fo})$ for any values of the space coordinate (ψ) and the time coordinate (Fo) is quite small and suitable for engineering calculations.

To find $\theta_{\lim}(\psi, \mathrm{Fo})$ we use the method of successive approximations. As the first approximation we can use the solution of the equations

$$\frac{\partial\theta_1(\psi, \mathrm{Fo})}{\partial \mathrm{Fo}} = \frac{\partial^2\theta_1(\psi, \mathrm{Fo})}{\partial\psi^2} + \frac{\Gamma-1}{\psi}\frac{\partial\theta_1(\psi, \mathrm{Fo})}{\partial\psi}, \tag{5}$$

$$\frac{\partial\theta_1(0, \mathrm{Fo})}{\partial\psi} = 0, \tag{6}$$

$$\frac{\partial\theta_1(1, \mathrm{Fo})}{\partial\psi} = \mathrm{Bi}_1[1-\theta_1(1, \mathrm{Fo})], \tag{7}$$

$$\theta_1(\psi, 0) = \theta_0, \tag{8}$$

where $\mathrm{Bi}_1 = \mathrm{Sk}(p + 4) = \mathrm{const}$.

It is obvious that because we have the relationship

$$\mathrm{Bi}_1 \geqslant \mathrm{Sk}[p + 1 + \theta(1, \mathrm{Fo}) + \theta^2(1, \mathrm{Fo}) + \theta^3(1, \mathrm{Fo})]$$

(the sign "=" pertains exclusively to the case in which $\mathrm{Fo} \to \infty$) the following condition will be satisfied:

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TABLE 1. Variation in Temperature at the Center and at the Surface of an Unbounded Plate (Sk = 2.0; Bi = 0; θ = 0.2; Γ = 1)

Fo	θ (0, Fo) according to the data [2]	θ (0, Fo) by the iteration method	δ, %	θ (1, Fo) according to the data [2]	θ (1, Fo) by the iteration method	δ, %
0,1	0,2146	0,2198	2,42	0,7548	0,7810	3,47
0,2	0,3012	0,3191	5,94	0,8391	0,8512	1,42
0,3	0,4087	0,4315	5,58	0,8774	0,8855	0,92
0,4	0,5070	0,5301	4,56	0,9026	0,9092	0,73
0,5	0,5910	0,6131	3,74	0,9215	0,9267	0,56
0,6	0,6614	0,6817	3,07	0,9364	0,9407	0,46
0,7	0,7201	0,7384	2,54	0,9483	0,9519	0,38
0,8	0,7689	0,7851	2,11	0,9579	0,9610	0,32
0,9	0,8093	0,8235	1,75	0,9656	0,9682	0,27
1,0	0,8428	0,8550	1,45	0,9719	0,9741	0,23

$$\theta_1(\psi, \mathrm{Fo}) \geqslant \theta(\psi, \mathrm{Fo}).$$

Here the sign "=" is valid only for the instant of time Fo = 0 and for Fo $\to \infty$.

Consequently, the temperature $\theta_1(\psi, \mathrm{Fo})$ can be treated as a limit value with respect to $\theta(\psi, \mathrm{Fo})$.

However, it is possible to approximate $\theta(\psi, \mathrm{Fo})$ more closely from above. Let us assume that we are required to find the change in temperature through the cross section of a body for a certain instant of time Fo = Fo*. We can then find a closer value than $\theta_1(\psi, \mathrm{Fo}^*)$ for the temperature, if we solve (1) under conditions (2) and (4), with the following boundary conditions at the surface (ψ = 1):

$$\frac{\partial\theta_2(1, \mathrm{Fo})}{\partial\psi} = \mathrm{Bi}_2[1 - \theta_2(1, \mathrm{Fo})], \tag{9}$$

where

$$\mathrm{Bi}_2 = \mathrm{Sk}\,[p + 1 + \theta_1(1, \mathrm{Fo}^*) + \theta_1^2(1, \mathrm{Fo}^*) + \theta_1^3(1, \mathrm{Fo}^*)] = \mathrm{const}.$$

It is probable that we can compile an inequality of the form

$$\theta_1(\psi, \mathrm{Fo}^*) > \theta_2(\psi, \mathrm{Fo}^*) > \theta(\psi, \mathrm{Fo}^*) \quad (\mathrm{Fo} \neq 0;\ \infty),$$

since we satisfy the condition

$$\mathrm{Bi}_1 > \mathrm{Bi}_2 > \mathrm{Sk}\,[p + 1 + \theta(1, \mathrm{Fo}^*) + \theta^2(1, \mathrm{Fo}^*) + \theta^3(1, \mathrm{Fo}^*)].$$

The subsequent iterations can be calculated in analogous fashion. As a rule, it is enough to limit ourselves to the third approximation, because higher-order approximations yield but insignificant refinements.

It should be noted that in each approximation step it is necessary to integrate the system of Eqs. (1)-(2) and (4) for the linear boundary condition

$$\frac{\partial\theta_j(1, \mathrm{Fo})}{\partial\psi} = \mathrm{Bi}_j[1 - \theta_j(1, \mathrm{Fo})], \quad j = 1, 2, \ldots, \tag{10}$$

where Bi_j is a constant which enables us to use rigorous analytical solutions [1].

In addition, the limit value for the temperature $\theta(\psi, \mathrm{Fo})$ can be approached from "below," if as the first approximation we take

$$\mathrm{Bi}_1 = \mathrm{Sk}\,(p + 1 + \theta_0 + \theta_0^2 + \theta_0^3).$$

The outstanding feature of this method lies in the fact that it makes it possible, rather simply, to establish the possible maximum calculation error. Thus, if Γ = 1, Sk(1 + p) = 2.0, p = 0, and θ_0 = 0.2, as demonstrated by calculations (Table 1) executed by a finite-difference method, the error in the determination of the temperature exhibits its greatest value at the initial instant of time for the thermal center of the item and it does not exceed 6%. For the surface of the body the overstatement is relatively smaller and amounts approximately to 3.5%. Table 1 gives the results of the third iteration. With an increase in the Fo number the divergence rapidly diminishes. The presence of a convection heat flow (p > 0) leads to a reduction in the difference between the limit temperature field and the actual field, and this is all the more pronounced, the greater the magnitude of the coefficient p.

The accuracy of the method is also raised by increasing the complex Sk(1 + p)(Sk(1 + p) > 2) and the initial relative temperature θ_0 of the body.

All other conditions being equal, the accuracy of determination for the temperature field in the case of a sphere and a cylinder is greater than in the case of a plate.

NOTATION

$\theta = T/T_m$	is the relative temperature;
$T(\psi, Fo)$	is the temperature of the body, °K;
T_m	is the temperature of the medium, °K;
$\psi = r/R$	is the relative coordinate;
R	is the characteristic dimension of the body (for the plate it is half its thickness, and for the cylinder and sphere it is the radius), m;
$Fo = a\tau/R^2$	is the Fourier number;
$Bi = \alpha R/\lambda$	is the Biot number;
$Sk = \sigma_a T_m^3 R/\lambda$	is the Stark number;
λ and a	are, respectively, the coefficients of thermal conductivity and thermal diffusivity, W/m·deg and m^2/h;
α	is the heat-transfer coefficient, $W/m^2 \cdot deg$;
σ_a	is the apparent heat-transfer coefficient for radiation, $W/m^2 (°K)^4$;
τ	is the time, h;
Γ	is the shape factor for the body, equal to 1 for a plate, equal to 2 for a cylinder, and equal to 3 for a sphere.

LITERATURE CITED

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2. A. V. Kavaderov and Yu. A. Samoilovich, Sb. Nauchnykh Trudov VNIIMT, No. 10, 14 (1963).